Assignment 5

R-4.5 Suppose we are given two *n*-element sorted sequences A and B that should not be viewed as sets (that is, A and B may contain duplicate entries). Give an O(*n*)-time pseudo-code algorithm for computing a sequence representing the set A ∪ B (with no duplicates).

Answer:

Algorithm union\_unique(A, B)  
 S 🡨 EmptySequene

if A.isEmpty() and B.isEmpty() then

return S

else if A.isEmpty() then

return B

else if B.isEmpty() then

return A

i 🡨 1, j 🡨 1

while j <=n and j <= n do {

v 🡨 0

p 🡨 A.atRank(i)

q 🡨 B.atRank(j)

if p.element() <= q.element() then

v = p.element()

i++

else

v = q.element()

j++

if S.isEmpty() or S.last().element() != v then

S.insertLast(v)

while i <= n do

p 🡨 A.atRank(i)

if S.isEmpty() or S.last().element() != p.element() then

S.insertLast(p)

i++

while j <= n do

q 🡨 B.atRank(j)

if S.isEmpty() or S.last().element() != q.element() then

S.insertLast(q)

j++

return S

R-4.9 Suppose we modify the deterministic version of the quick-sort algorithm so that, instead of selecting the last element in an *n*-element sequence as the pivot, we choose the element at rank (index) ⎣*n*/2⎦, that is, an element in the middle of the sequence. What is the running time of this version of quick-sort on a sequence that is already sorted?

Answer: If the middle element of a sorted sequence ‘S’ is selected as a pivot then,

-Size of both Lower Partition (L) and Greater partition (G) will be always at least S/4.

-So the height of the quick-sort tree will be **log4/3n**

-The running time for each depth is **O(n)**

**-Therefore,** total running time of quick sort will be **O(nlog4/3n)🡪O(nlogn).**

C-4.10 Suppose we are given an *n*-element sequence S such that each element in S represents a different vote in an election, where each vote is given as an integer representing the ID of the chosen candidate. Without making any assumptions about who is running or even how many candidates there are, design an *O(n* log *n)*-time algorithm to see who wins the election S represents, assuming the candidate with the most votes wins.

Answer:

Algorithm count\_duplicate(S)

if S.isEmpty() then

throw EmptySequeneException

lo 🡨 S.rankOf(S.first())

hi 🡨 S.rankOf(S.last())

id 🡨 S.first().element()

cnt 🡨 1

custom\_quick\_sort(S, lo, hi, id, cnt)

return id;

Algorithm custom\_quick\_sort(S, lo, hi, id, cnt)

if lo < hi then

p 🡨 custom\_partition(S, lo, hi, id, cnt)

custom\_quick\_sort(S, l, p-1, id, cnt)

custom\_quick\_sort(S, p+1, hi, id, cnt)

Algorithm custom\_partition(S, lo, hi, **id, cnt**)

p 🡨 random(lo, hi)

S.swapElements(S.atRank(lo), S.atRank(p))

**pi 🡨 S.atRank(lo).element()**

**t = 1**

j 🡨 lo+1

k 🡨 hi

while j <= k do {

while k >= j and S.atRank(k).element() >= pi do

if S.atRank(k).element() = pi then

**t++**

k--

while j <= k and S.atRank(j).element() <= pi do

if S.atRank(j).element() = pi then

**t++**

j++

if j < k then

S.swapElements(S.atRank(j), S.atRank(k))

}

S.swapElements(S.atRank(lo), S.atRank(k))

**if t > cnt then {**

**id = pi**

**cnt = t**

**}**

return k

Modify the algorithm inPlaceQuickSort to handle the general case efficiently when the input Sequence S may have many duplicate keys. Hint: partition into three segments, the first segment containing keys less than the randomly selected pivot (unsorted), the second containing keys equal to the pivot, and the third segment containing keys greater than the pivot (unsorted). The inPlacePartition will return two indices (p1, p2) where p1 is the index of the first element equal to the pivot and p2 is the index of the last element equal to the pivot; the elements equal to the pivot are in sorted order, that is, all elements equal to the pivot will be in their sorted position within the Sequence. You are to write a pseudo code algorithm for inPlacePartition. The top-level of the algorithm that calls inPlacePartition will be as follows:

Algorithm inPlaceQuickSort(S, lo, hi)

**Input** Sequence S, ranks lo and hi represent a segment of the Sequence S to sort

Output Sequence S with the elements with rank between lo and hi rearranged in sorted order

if lo<hi then

(p1,p2) ← inPlacePartition(S, lo, hi) // keys between p1 and p2 are sorted

inPlaceQuickSort(S, lo, p1 - 1)

inPlaceQuickSort(S, p2 + 1, hi)

Answer:

Algorithm inPlacePartition (S, lo, hi)

L, E, G 🡨 EmptySquence

p 🡨 random(lo, hi)

S.swapElements(S.atRank(lo), S.atRank(p))

pi 🡨 S.atRank(lo).element()

j 🡨 lo+1

k 🡨 hi

while j <= k do {

while k >= j and S.atRank(k).element() >= pi do {

if S.atRank(k).element() = pi then

E.insertLast(S.atRank(k))

else

G.insertLast(S.atRank(k))

k--

}

while j <= k and S.atRank(j).element() <= pi do {

if S.atRank(j).element() = pi then

E.insertLast(S.atRank(j))

else

L.insertLast(S.atRank(J))

j++

}

if j < k then

S.swapElements(S.atRank(j), S.atRank(k))

}

S 🡨 L ∪ E ∪ G

p1 🡨 L.size()+1

p2 🡨 p1 – 1 + E.size()

return Pair(p1, p2)

Let L be a **List** of objects colored either red, green, or blue. Design an **in-place** algorithm **sortRBG(L)** that places all red objects in list L before the blue colored objects, and all the blue objects before the green objects. Thus the resulting List will have all the red objects followed by the blue objects, followed by the green objects. **Hint:** use the method swapElements to move the elements around in the List. **To receive full credit**, you must use positions for traversal, e.g., first, last, after, before, swapElements, etc. which is necessary to make it in-place.

Answer:

Algorithm sortRBG(L)

if L.isEmpty() then

return

p 🡨 L.first()

while !L.isLast(p) do

p 🡨 L.after(p)

if p.element = R then

shiftLeft(L, p)

if p.element = G then

shiftRight(L, p)

Algorithm shiftLeft(L, p)

if L.isFirst(p) then

return

q 🡨 L.before(p)

L.swapElements(p,q)

shiftLeft(L, p)

Algorithm shiftRight(L, p)

if L.isLast(p) then

return

q 🡨 L.after(p)

L.swapElements(p,q)

shiftRight (L, p)